HYDRODYNAMICS OF PLATE COLUMNS. VII.*

VERIFICATION OF AN EXPERIMENTAL METHOD FOR STATISTICAL ANALYSIS OF THE DYNAMIC PROPERTIES OF GAS-LIQUID MIXTURE

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Received December 14th, 1970

An experimental method of pickup and evaluation of random pressure fluctuations has been worked out enabling direct determination of their statistical characteristics. A requested accuracy at reasonable time costs is achieved by collecting the output signals on a magnetic tape with subsequent conversion to the digital form on a Dynamco data logger, or by a direct analog computation on a Musa 6 statistical analyzer. The calculation of the estimates of given characteristics, their comparison and examination of statistical equivalence was made for experiments carried out on a model equipment under some typical regimes. The quantities obtained, together with the calculated cross-correlation functions, will form the basis for further analysis of the process on a sieve plate and formulation of the model.

The foregoing experiments[1] on a 120 mm in diameter column consisting in recording the pressure fluctuations on a photographic film have revealed that the character of random fluctuations in the space between the plates necessitates observation in time intervals of the order 100 s and the size of samples equalling approximately 10^4. Under these conditions and anticipated extent of further systematic experiments this method appears too time consuming and impracticable. Apart from a possibility of applying auxiliary statistical analyzers[2], the recording of a random process on a magnetic tape – commonly used in statistical dynamics – appears convenient. Since we are dealing with random processes exhibiting spectra from the low frequency range (0–30 Hz), the use of a special instrumentation tape recorder seems necessary, or, if that is not available, an adapted commercial tape recorder. With respect to the nature and extent of further research, the latter alternative seems more realistic. The calculation of characteristics of the random process may then be carried out after conversion of the record to digital form on a digital computer, eventually, without digitalization, on an analog correlator. The application of both alternatives permits one to assess the error caused by the finite density of sampling of the signal at digital processing, eventually other errors of both methods. A repeated recording and

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* Part VI: This Journal *36*, 1753 (1971).

computation of statistical characteristics of the same random process will provide the insight about the reproducibility of each approach and hence about random errors. Existing statistical tests enable to judge whether statistical characteristics obtained may be regarded as statistically equivalent and as such be averaged to obtain smoothed values.

A scheme of experiments selected for verification was deviced with future utilization of the results in mind.

THEORETICAL

The definitions of statistical quantities characterizing stationary, ergodic random process, *i.e.* the mean, the autocorrelation function, the variance, the standard deviation and the spectral density, as well as their estimates for finite time intervals were given in Eqs (*1*)–(*4*), respectively (*5*)–(*8*) of the preceding paper[1]. In this place we want to give in detail some limitations and specificities necessary for a concrete application of general definitions, particularly with regard to the conducted statistical tests. In determining the mean and the standard deviation of the pressure fluctuations we are interested primarily in their dimensional form, *i.e.* the magnitude in units of pressure. Any comparison of two values of this category carries inevitably the error of calibration of the measuring device which depends on the absolute value of the measured quantity. This may misrepresent the statistical tests. For the spectral density and the autocorrelation function expressed in normalized, *i.e.* dimensionless, form this drawback is without effect and the conversion by means of the calibration data is needless. Thus it may be expected that the statistical tests will be undistorted. The reliability of determination of the standard deviation will therefore be judged expediently only by comparing the values obtained by different methods, respectively those from different measurements. The degree of agreement may be compared with the anticipated maximum relative error $\pm 10\%$ which follows for most of the evaluated records from considerations of the preceding paper[1].

The consistency of the spectral densities (and hereby the autocorrelation functions) and statistical equivalence of the spectral densities determined by both methods for the same statistical random process and by one method for two independent records of the same process is determined by means of statistical tests[3] presented subsequently.

Nyquist's frequency of the digital method, 16 Hz, determines the upper limit of frequencies, f_c, which can be detected in the spectrum of the processes studied. Higher frequencies than f_c appear aliased with a frequency $0 \leqq f \leqq f_c$ according to the relation[3] as $(2f_c \pm f)$, $(4f_c \pm f)$, $(2n \,.\, f_c \pm f)$. This fact must be kept in mind when evaluating spectral densities of some of the pressure charts recorded under the bubbling regime. A criterion of the real existence of the frequencies found in the spectrum is the spectral density determined by the analog method with Nyquist's frequency of 158 Hz.

A suitable test of consistency of the calculated spectral densities is afforded by equation

$$\int_{-\infty}^{\infty} g^{**}_{\mathrm{PP}}(\omega)\,\mathrm{d}\omega = 1{\cdot}0 \qquad (1a)$$

following from definition of the spectral density.

For a physically realizable spectrum, this equation takes the form

$$\int_{0}^{\infty} g^{**}_{\mathrm{PP}}(\omega)\,\mathrm{d}\omega = 0{\cdot}5\,. \qquad (1b)$$

We worked with the values of the spectral densities smoothed by weighting function – the spectral window – at the Fourier transform of the autocorrelation function in the time interval $(0, \tau_{\max})$. Thus

$$G^{**}_{\mathrm{PP}}(\omega) = 2\int_{0}^{\tau_{\max}} R^{**}_{\mathrm{PP}}(\tau)\,D(\tau)\cos(2\pi\omega\tau)\,\mathrm{d}\tau\,, \qquad (2)$$

where Hamming's spectral window of the width $\tau_{\max}$ used here is defined by equation

$$D(\tau) = 0{\cdot}54 + 0{\cdot}64\cos(\pi\tau/\tau_{\max})\,. \qquad (3)$$

The obtained estimates of the spectral density $G^{**}_{\mathrm{PP}}(\omega)$ may be regarded as quantities with the normal distribution for the degree of freedom $m \leqq 30$, where[3]

$$n = 2B_{\mathrm{e}}T_{\mathrm{r}} = 2T_{\mathrm{r}}/\tau_{\max}\,. \qquad (4)$$

It can be derived that two estimates of the spectral density $[G^{**}_{\mathrm{PP}}(\omega)]_1$, $[G^{**}_{\mathrm{PP}}(\omega)]_2$ of the same process in the frequency interval $(\omega_{\mathrm{A}}, \omega_{\mathrm{B}})$ of the width $B = \omega_{\mathrm{B}} - \omega_{\mathrm{A}}$ may be regarded as being statistically equivalent on the significance level α, if the value of the following quantity

$$D = [N_{\mathrm{f}}(2/n_1 + 2/n_2)]^{-1/2}\sum_{i=1}^{N_{\mathrm{f}}}\log\{[G^{**}_{\mathrm{PP}}(\omega)]_1/[G^{**}_{\mathrm{PP}}(\omega)]_2\}\,, \qquad (5)$$

where

$$N_{\mathrm{f}} = B/B_{\mathrm{e}} \qquad (6)$$

satisfies inequality given by the critical values of the normal distribution, *i.e.*:

$$-Z_{\alpha/2} \leqq D \leqq Z_{\alpha/2}\,. \qquad (7)$$

EXPERIMENTAL

A hydrodynamic model 300 mm in diameter described in detail in the preceding paper[4] was used for experiments. The diameter of the openings in the sieve plate without downcomer was 6 mm and the free area of the plate was 15%. For a given liquid flow rate, the gas flow rate was properly selected according to the in advance known operation characteristic so as to obtain gradually the regime of bubbling, regime of mobile froth and the oscillatory regime. At these flow rates the pressure was recorded at the level of the plate (*P*1), above the plate (*P* 3) and below the plate (*P* 2). A block diagram of the measuring equipment is shown in Fig. 1. The knowledge of statistical characteristics of these pressures and their mutual correlation functions permits calculation

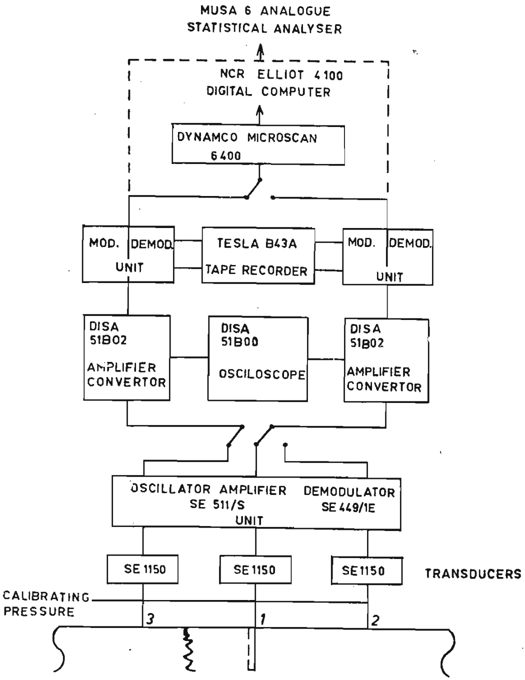

FIG. 1

Block Diagram of Experimental Set-Up

of statistical characteristics of pressure differences across the opening, froth and the plate, which are obviously controlling in the operation of the plate, as well as the mutual relation of these pressures in the amplitude and time domain. Pressure was detected by means of inductance transducers[5] SE 1150 in the range 2500 to 6250 Nm^{-2}. The variation of inductance modulated a high frequency voltage (3 kHz) generated by an SE 511/S oscillator[5] located in a common block together with six demodulator-amplifier[5] units of the SE 449/1 E type. The maximum DC output was $\pm 1{\cdot}8$ V. In order that the whole frequency swing of the adapter and the tape recorder was made full use of, the DC signals were further amplified by means of stable Disa amplifiers built in 51 B O2 units. A parallel arrangement allowed then a pickup of pairs of signals by means of the mentioned adapters with a frequency modulator designed in ČSAV[6] and collection onto two tracks of a semi-professional stereo tape recorder Tesla B 43. In addition, the detected signals were brought onto the picture tube of a 51 B OO oscilloscope. At digital evaluation, the signals from magnetic tapes were individually (one track at a time) fed with or without a twofold reduction of speed of the record through the adapter into one channel of a Dynamco 6400 data logger. Here the signal was sampled by a frequency of 16 Hz with the digital output on an 8-track punched paper tape. Nyquist's frequency at the twofold speed reduction is thus 16 Hz. The data were then processed on an Elliot 4100 computer. At the analog evaluation, the data were fed by pairs into a universal statistical analyzer Musa 6. The manufacturer guarantees high stability of the input units of the measuring system (transducers, mod. and demod. units 0·02%/°C) and linearity (transducers 0·2%, amplifier, demodulator 0·1%). For the output units too (adapter: instability 0·1%, linearity 0·4%). Manufacturer supplied characteristics are flat in low frequency range (mod., demod. 0—500 Hz ± 1 dB, adapter 0—850 Hz — 3 dB). Controlling, however, is the frequency characteristic of the transducer which was not provided by the manufacturer. That may, of course, be affected by the length of the column-transducer line as well as by the medium within it, *i.e.* either water in transducer *1*, or air in transducers *2*, *3*. In order that the frequency characteristic of the transducers could be assessed, their response to a unit jump was investigated with direct connection and/or with a different length and diameter of the connection tubings filled with either water or air. It has been established that the length of the connecting tubing in a given range 50—150 cm does not affect the frequency characteristics of the used type of the transducer. From the calculated time constant of a 100 cm long (5 mm in diameter) polyethylene connecting tubing filled with water results by a first order approximation a frequency characteristic flat in the range 0—20 Hz (—2 dB for 12 Hz). For the same connecting tubing, 150 cm long, filled with air in the interval 0—30 Hz (—2 dB for 22 Hz). Thus we can expect the amplitude and phase distortion in the anticipated frequency range[1] to be relatively small, particularly under the regimes important for operation of the column in practice. The calibration of the whole system by static pressures in the whole operating range of the transducer has shown the total nonlinearity less than 2%.

RESULTS

A total of four experiments were performed at the selected liquid flow rate 0·34 kg/m^2s and the gas velocity equalling $w_G = 0{\cdot}15, 0{\cdot}35, 0{\cdot}60, 1{\cdot}82\ m\ s^{-1}$. Under conditions of experiment *1* the regime of bubbling prevailed on the plate, the regime of mobile froth in experiments *2*, *3* and the oscillatory regime in experiment *4*. Combinations of random pressures $P1-P2$, $P1-P3$, $P2-P3$ were recorded in every experiment, duration of each being 4 to 5 minutes. All records were digitalized (the size of the collected sets was 4000 to 5000 data) and processed by the already mentioned digital method[1]. In addition, the analog processing was performed, in which sections of the record of standard length 180 s were evaluated. Altogether, we processed 24 records by the analog and the digital methods. The digital calculation of the autocorrelation function was carried out for time increments of the length of one sample, *i.e.* 32 ms to $\tau_{max} = 2$ s; the analog calculation for time increments 9·16 ms to $\tau_{max} = 2$ s. Corresponding width of the spectral window is thus $1/\tau_{max} = 0{\cdot}5$ Hz.

Table I gives a comparison of the times T_4, T_5 necessary for attaining the 1% relative error of determination of the mean and the standard deviation calculated on the basis of the estimate of the autocorrelation function with the length of evaluated time intervals T_r. It comes as no surprise that the lengths of interval suffice for determining the mean with requested accuracy; that could be inferred already from

TABLE I

Comparison of Times T_4, T_5 with the Length of Time Interval T_r

Experiment	Pressure	T_4	T_5	T_r	Integral in Eq. (*1b*) digital method	Integral in Eq. (*1b*) analog method
1	*P* 2	23	63	145	0·4902	0·5071
1	*P* 3	11	1 420	145	0·4986	0·5329
1	*P* 1	1	9	129	0·5011	0·4324
1	*P* 3	18	1 200	145	0·4977	0·5005
1	*P* 1	1	16	145	0·4801	0·4308
1	*P* 2	28	78	145	0·7980	0·4226
2	*P* 1	1	12	145	0·4902	0·4782
2	*P* 2	3	1 260	121	0·5124	0·4097
2	*P* 1	1	18	145	0·4942	0·4744
2	*P* 3	4	1 260	145	0·4958	0·4949
2	*P* 2	3	1 400	145	0·5239	0·6170
2	*P* 3	8	1 200	145	0·5063	0·4997
3	*P* 2	3	11	129	0·4977	0·5162
3	*P* 3	11	83	129	0·4931	0·4980
3	*P* 1	4	13	121	0·4944	0·5212
3	*P* 3	8	95	129	0·4989	0·5037
3	*P* 1	3	16	129	0·4935	0·4346
3	*P* 2	2	10	129	0·4839	0·4509
4	*P* 1	1	15	121	0·4958	0·4570
4	*P* 2	2	63	121	0·4946	0·4792
4	*P* 1	1	17	121	0·4861	0·4653
4	*P* 3	11	1 260	121	0·4989	0·5007
4	*P* 2	1	51	121	0·4851	0·4664
4	*P* 3	12	1 000	115	0·5156	0·4983

the findings of the preceding paper[1]. In most cases these intervals suffice for determining the standard deviation with this accuracy too. If it is not the case, as particularly for pressures $P2$ and $P3$, the difference between the times T_5 and T_r is smaller in the order of magnitude than that found before. Here as well the times T_5 represent an approximation based on the estimate of the autocorrelation function in a limited time interval.

The degree of accuracy achieved in determination of the standard deviation by the digital method is evidenced in Fig. 2 plotting the standard deviation of pairs of data for all experiments, a total of 12 points. It can be concluded that the accuracy achieved comports well with the anticipated relative error ±10%.

Similar comparison was made with the results of the analog method yielding the same conclusion (Fig. 3). A comparison of corresponding pairs of values of the standard deviation obtained by the analog and the digital method, a total of 24 points, exhibits a larger discrepancy (Fig. 4). The values calculated by the analog method are consistently higher than those calculated by the digital method. One can thus infer on a systematic error given by the difference of both methods. Even in this case, however, the accuracy achieved may be regarded as satisfactory.

The consistency of calculation of the spectral densities for both the analog and the digital method is apparent from Table I summarizing the values of the integral in Eq. (*3*) calculated by the trapezoidal rule with upper limit $\omega = 30$ Hz. The average deviation from the theoretical value 0·50 was 1·7% at the digital evaluation and 6·7% at the analog evaluation. These values suggest that the consistency of the digital method is higher than that of the analog method. This is attributed primarily to the

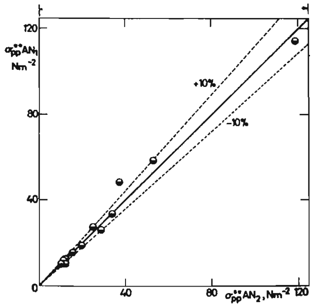

FIG. 2

Comparison of Standard Deviations Found at the Analog Evaluation

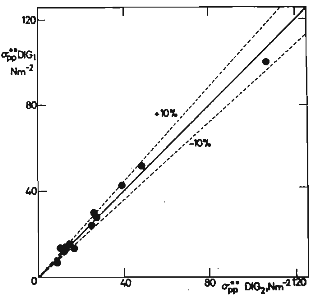

FIG. 3

Comparison of Standard Deviations Found at the Digital Evaluation fact that for the digital method the Fourier transform of the autocorrelation function takes place directly from the values stored on the computer memory, *i.e.* with unlimited accuracy, while for the analog method the autocorrelation function is obtained in a graphical form and for the subsequent Fourier transform the latter must be converted into digital form by reading off with considerably limited accuracy. Further, the values in Table I suggest that the consistency of the spectral densities *P*1 and *P*3 is higher than that of *P*2; the average relative deviations are respectively 0·017, 0·010, 0·024 for the digital method and 0·087, 0·012, 0·108 for the analog method.

Table II summarizes the results of the statistical test in Eq. (*3*). Both the analog an the digital method were tested at the significance level $\alpha = 0{\cdot}01$, *i.e.* the result of the test is positive if $-2{\cdot}58 \leqq D \leqq 2{\cdot}58$. Analysis of results of the test for the digital method shows that the test was satisfied with exception of *P*2 for experiment *1* where owing to an unknown failure a distortion of the signal occurred during

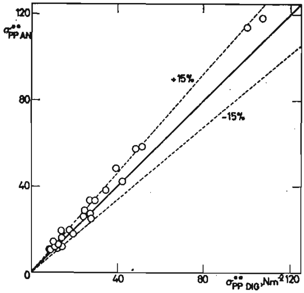

FIG. 4

Comparison of the Analog and Digital Evaluation Method

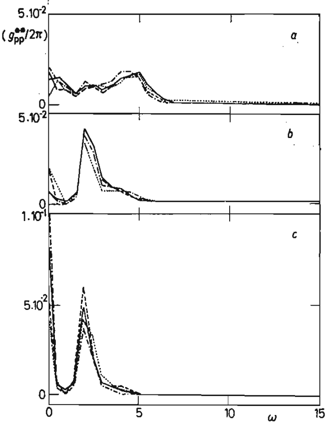

FIG. 5

Comparison of Calculated Normalized Spectral Densities $g^{**}_{PP}(\omega)$ for Experiment *4*, Spectral Window 0·5 Hz, ω(Hz)

Pressure *a* *P*1, *b* *P*2, *c* *P*3; method ———, ... digital; — — —, —·—·— analog.

conversion to the punched tape form. It may be concluded that the spectral densities $P1$ and $P3$ display statistical equivalence to a greater extent than $P2$. This can be partly attributed to the fact that for the pairwise pickup of pressure $P2$, as the only one, two different transducers were used.

A comparison of statistical equivalence of the spectral densities determined by the analog and the digital method is seen from Table III. Here too, a majority of results of the test are positive. With respect to the mentioned aliasing of frequencies the region of intermediate frequencies, which are physically nonexistent in the spectrum, was omitted from the test. The hypothesis of statistical equivalence was rejected in four cases out of the total of 23, primarily for $P3$.

A more detailed analysis of results of the test with the reference to the real shape of the calculated spectral densities shows that the least agreement is found in the region of very low frequencies, mainly for $f = 0$ Hz. This is apparent also from Fig. 5 showing normalized spectral densities $P1$, $P2$, $P3$ for experiment *4* – the oscillatory regime of the plate. This fact, reported also in the literature[3-8], affects strongly the results of the test for the whole investigated frequency range. Moreover, small absolute values of spectral densities near the upper limit of the frequency range studied carry greater errors which in turn affect the over-all result of the test. Restriction of the tested region of frequencies to vicinity of important peaks, which was in many cases put in effect, improved the results of the test.

TABLE II

Consistency of Calculation of the Spectra Densities

Experiment		Digital method			Analog method		
		frequency range, Hz	D	result	frequency range, Hz	D	result
1	$P1$	0–25	0·65	+	0–25	2·56	+
1	$P2$	not evaluated			0–15	0·24	+
1	$P3$	0–15	0·70	+	0–15	2·27	+
2	$P1$	0–25	1·39	+	0–25	0·57	–
2	$P2$	0–25	2·39	+	0–25	1·83	–
2	$P3$	0– 6	0·92	+	0– 6	0·39	+
3	$P1$	0–15	0·68	+	0–15	2·00	–
3	$P2$	0–15	1·33	+	0–15	3·34	–
3	$P3$	0– 6	1·84	+	0– 6	2·09	+
4	$P1$	0– 8	0·16	+	0– 8	1·52	+
4	$P2$	0– 6	0·58	+	0– 6	0·16	+
4	$P3$	0– 6	2·44	+	0– 6	1·84	+

TABLE III
Comparison of Statistical Equivalence of the Spectral Densities

Experiment	Pressure	Frequency range	*D*	Result
1	*P* 2	0—15	not evaluated	
1	*P* 3	0—15	6·20	−
1	*P* 1	0—6, 17—25	2·38	+
1	*P* 3	0—15	1·89	+
1	*P* 1	0—6, 17—25	2·20	+
1	*P* 2	0—15	0·29	+
2	*P* 1	0—7, 17—23	2·35	+
2	*P* 2	0—6, 21—25	0·74	+
2	*P* 1	0—7, 17—23	2·31	
2	*P* 3	0— 6	2·32	+
2	*P* 2	0—6, 21—25	2·83	−
2	*P* 3	0— 6	1·12	+
3	*P* 2	0—15	0·39	+
3	*P* 3	0—15	5·20	−
3	*P* 1	0—15	0·47	+
3	*P* 3	0— 6	1·21	+
3	*P* 1	0—15	0·66	+
3	*P* 2	0—15	1·94	+
4	*P* 1	0— 8	1·68	+
4	*P* 2	0— 6	0·71	+
4	*P* 1	0— 8	1·91	+
4	*P* 3	0— 6	2·71	−
4	*P* 2	0— 6	1·65	+
4	*P* 3	0— 6	2·49	+

The experimental verification has shown suitability of the used experimental method of evaluation of random pressure fluctuations in the whole operation range of a model sieve plate without downcomer. Analysis of evaluated independent realizations of the same random process shows objectively a good reproducibility for both alternatives of evaluation. The evaluation of one realization by both methods has shown a good agreement too. Assuming the predominance of application of the digital method of evaluation (availability), Nyquist's frequency has to be increased owing to aliasing of the frequencies primarily under the regime of bubbling. A possible solution to the problem is the fourfold speed reduction at digitalization. The credibility of the results obtained from check experiments enables their future use in analysis of the mechanism of operation of a sieve plate without downcomer.

LIST OF SYMBOLS

B	width of frequency interval
B_e	equivalent width of interval
D	quantity in Eq. (*3*)
$D(\tau)$	spectral window
$G_{PP}^{**}(\omega)$	estimate of the spectral density
$g_{PP}^{**}(\omega)$	estimate of the spectral density in normalized form
n	number of degrees of freedom
P	pressure
$R_{PP}^{**}(\tau)$	estimate of the autocorrelation function
T_r	evaluated time integral
Z	quantity with normal distribution
$\tau_{\max}$	maximum time delay
ω	frequency, Hz
δ	standard deviation
DIG	digital
AN	analog

Subscripts

1,2	run number

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Translated by V. Staněk.